

Grade 12 — Vectors, Three-Dimensional Geometry & Determinants

Question 1

If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, and $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$, find the angle between $\vec{a}$ and $\vec{b}$.

Question 2

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$, and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} + \vec{b} = \vec{0}$ and $\vec{a} \cdot \vec{c} = 3$. Find $\vec{c}$.

Question 3

Show that the vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ are coplanar if and only if $\vec{a} + \vec{b}$, $\vec{b} + \vec{c}$, $\vec{c} + \vec{a}$ are coplanar. Prove your result using the scalar triple product.

Question 4

Find the shortest distance between the lines:

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$

$$\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Also state, with justification, whether the two lines intersect.

Question 5

A line makes angles α , β , γ , δ with the four diagonals of a cube. Prove that:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$$

Question 6

Find the image (reflection) of the point $(1, 2, -3)$ in the line

$$(x + 1)/2 = (y - 3)/(-2) = z/(-1)$$

Question 7

Using properties of determinants, prove that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

Question 8

Show that the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

satisfies the matrix equation $A^2 - 4A - 5I = O$, and hence find A^{-1} .

Question 9

Using determinants, find the area of the triangle whose vertices are $(3, 8)$, $(-4, 2)$, and $(5, -1)$. Then, treating these points as position vectors in the xy -plane ($z = 0$), determine the value(s) of k for which the point $(k, 0, 0)$ is collinear with any two of the given points, using the scalar triple product condition for collinearity.

Question 10

Find the angle between the two diagonals of a cube of side a .

Question 11

A line with direction cosines proportional to $2, 1, 2$ meets each of the lines $x = y + a = z$ and $x + a = 2y = 2z$. Find the coordinates of each of the points of intersection.

Question 12

Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$, measured parallel to the line $x/2 = y/3 = z/(-6)$.